Formulation of Microsecond and Sub-microsecond Fluctuations in Geomagnetic Field due to Lightning

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ABSTRACT

Intracloud, positive and negative CG-lightning, exhibit microsecond and sub-microsecond scale waveforms in the in-situ electric field with average angular frequency (ω) ≈ O (103 to 105); sometimes even as high as ≈ O (107) due to the 'lightning-bursts'. The electric field fluctuations induce corresponding variability in ensuing induced magnetic field - though not identically. Hence spontaneous fluctuations are expected in total magnetic field, too. Fluctuations in induced magnetic field may be formulated as B = B0Cos(ωt) + BB. The horizontal component of the concentric circles of induced magnetic field interferes with in-situ linear geomagnetic field(Gm) and causes corresponding micro and sub-microsecond fluctuations which could be formulated as B = B0Cos(ωt) + BBm (where BBm = BB + Gm ; a vectorial addition). There will be a diameter-axis for the azimuthal induced magnetic field concentric circles, interfering with the in-situ geomagnetic field, where optimum fluctuations (maximum in one half and minimum in other half) will be experienced. The formulation has its application within corona envelope wherein the Lorentz force has to be accounted as an additional body force during the magnetohydrodynamic analysis of the atmospheric momentum equations.

Keywords: *Micro and Submicrosecond Fluctuations, Geomagnetic Field, IC/CG Lightning, Continuing Current, Lorentz force and Corona envelope.*

1. Introduction

Historically Grenet (1947) proposed first "convective" mechanism theory of lightning which was supported by Vonnegut (1953). Subsequently Sartor (1973) had described necessary conditions for cloud electrification as coexistence of precipitation size particles and small ice crystals, both in relatively high concentration, associated with temperature sufficiently high for polarization charge transfer to be efficient and fairly strong updraft to slow down the passage of larger charge transfer-efficient particles through the limited fast relaxation zone. Convection currents cause strengthening of the electric field in the cloud (Sounders, 1993). It is now well accepted that thunderstorm charge generation is the product of hydrometeor collisions in the presence of supercooled liquid water (noninductive or relative growth rate - Saunders et al., 2006) which produces lightning. It can happen in towering cumulus prior to cumulonimbus (Benjamin et al., 2019). Brief details of positive, negative and intracloud lightings and related issues are categorically presented in the following subsections for completeness.

1.1 Positive CG lightning

Positive is about 10 times stronger than the average current of negative CG lightning and accounts only 10% of total CG lightnings (Uman 2011, Rakov 2003). Out of 52 positive cloud-to-ground flashes studied by Nag and Rakov(2012) 42 (81%) were single-stroke, 9 (17%) were two-stroke, and only 1 (2.0%) was three-stroke flashes. They inferred that 3 subsequent strokes in their data had followed the previously created (first-stroke) channel. Thus multiple stroke positive flashes are relatively rare and are mostly composed of a single stroke (Saba et al. 2010). Out of 36 positive flashes 32 contained one stroke and four contained two strokes (Heidler et al., 1998). Wu et al.(2020) had observed mean multiplicity of 1.24 and maximum of 5 for the positive strokes.

Several charge structures within cloud have been presented in literature as cause of positive CG lightning. Brook et al.(1982) had proposed that the exposing of positive charges to ground, due to vertical wind shear was the cause of positive lightning, whereas existence of positive monopolar charge structure was given by Kitagawa and Michimoto (1994). Orville and Huffines (2001) found inverted dipole (negative charge region above positive charge region in cloud) as the cause of positive CG lightning. Cui et al. (2005, 2009) inferred tripolar charge structure i.e. positive at the top, negative in the middle, and an additional positive below the negative. Positive CG lightning when a negative charged horizontal leader is cut-off from the older channel and the newly formed rear end of this leader gets positively charged then a positive leader forms at the rear end of the advancing negative leader channel causing a positive cloud-to-ground flash. Positive cloud-to-ground discharges produced by branching of in-cloud discharge channels, probably most often when these channels occur near or below the cloud base. It has been observed to occur with the frequency of more than 1.5 min-1 prior to tornadogenesis by Bluestein and MacGorman (1999).

1.2 Negative CG lightning

90 – 95% of all the CG lightning are Negative CG lightning and they are relatively much weaker than the positive lightning. On an average negative CG lightning produces 30 kA current. Krehbiel (1986) had reported that the lower negative charge region in a thunderstorm is normally steady at the altitude around 7 km corresponding to a temperature around −15◦C. Centre height of the main negative charge region increases with increasing average balloon ascent rate and updraft speed at a rate of about 0.3 km per 1 ms- 1, with a correlation coefficient of 0.94 (Maribeth stolzenburg et al.1998). Cloud charge configuration is normally 'Vertical negative dipole'. Therefore the negative charges are accumulated near lower part of the cloud, facing ground. About 80% of negative flashes contain two or more strokes (Nag and Rakov 2012). Ariadi (2016) observed negative CG flash may consists of 'on an average 5.2 return strokes' which could be even as high as ≈18 sometimes. Normally observed time to complete a return stroke is 100-200 microsecond (Uman 2011).

1.3 Intracloud (IC) lightning

When discharge occurs between areas of differing electric potential within a single cloud, it is known as intra-cloud (IC) lightning. Kitagawa and Brook (1960) divided the cloud-discharge field change into three portions i.e. an initial portion, a very active portion, and a later or junction (J)-type portion. Within first 1 ms duration, the initial breakdown process of IC discharge has relatively less than half number of pulses, less bipolar pulses than unipolar pulses and larger inter-pulse duration when compared to cloud to ground discharge. While lightning flashes to the ground are characterized by rapid return-stroke field changes occurring every 50 ms or so and lasting for the time-period of the order of 1 ms and cloud discharges produce slow, relatively smooth field changes. Cloud(IC) and cloud-to-ground(CG) discharges have about the same total time duration, generally a fraction of a second. Electrical records reveals that the late stages of intracloud discharges are very similar to those of cloud-to-ground discharges during the periods between successive return strokes (junction process) and during the period after the last return stroke (final process). Based on the Earth Networks Total Lightning Network (ENTLN) 2017-2019 data the mean IC current recorded in India was 8-10 kA; though it may range from 1 to 30 kA. It has been observed by Nag and Rakov (2009a) that often presence of excessive lower positive charge region may even prevent the occurrence of negative CG-discharge by blocking the descending negative leader and thus converts the negative CG-flash to intracloud (IC) or cloud-to-air flash.

1.4 Polarity reversal

Lightning current waveforms exhibiting polarity reversal were initially reported by McEachron(1939,1941). Polarity reversal from positive to negative CG lightning is also observed in context of tornadoes (Branick and Doswell, 1992). Curran and Rust (1992) observed a splitting supercell thunderstorm and noted that polarity reversal followed it during CG lightning prior to tornado formation. Bipolar lightning discharge to ground also causes the flipping of azimuthal induced magnetic field direction due to reversal of polarity and is one of the causes of transient-fluctuation of resultant horizontal magnetic field in association of geomagnetic field.

2. Corona Envelope, Electrical Conductivity (σ) and Induced Magnetic Field

Heckman and Williams (1989) presented schematic width of corona envelope of about ≈ 120 m radius in clear air around a vertical negative CG lightning where positive ion streamer will radially propagate. They said that the limit of radial propagation, however, restricts to only 100m or 90m in diluted cloud (liquid water content of 0.5g/Kg) or dense cloud ((liquid water content of 5g/Kg). Few millisecond prior to formation of subsequent return stroke in the dart leader positive streamers push out to 90m then they slowly drift away at 20 m/s to another 2m (Maslowski and Rakov 2006). Hence shape the corona envelope for return stroke looks like inverted cone's segment. Its height is limited to bridging point of upward streamer with downward leader.

Electrical conductivity of the lightning channel is in the range of (1.62–2.27)×104 S m−1 (Guo et al., 2009). Marjanovic and Cvetic (2009) used generalized lightning travelling current source(GTCS) return-stroke model to examine the electrical-conductivity(σ) of lightning channel corona sheath(space in between distance ≈ 1.5cm and 6.0cm from outer periphery of core). They found minimum conductivity of about 10 µS/m in a close annular cylindrical space. Interestingly, they noted that σ value rises away from centre for some distance. It is well known that the rising value reverses and reaches to 10-8 µS/m far away (≈ ≥ 150m) in clear air (Uman 2011). In view of the corona envelope (Heckman and Williams,1989; radius ≈ 100m) dimension it may be assumed in the present study that at 1m radial distance from channel-core σ = 10 µS/m and between 10 - 50m distance the σ ≈ 1µS/m. Within this domain the significance of Lorentz force as one of the body forces cannot be disdained and hence magnetohydrodynamic analysis of the atmospheric motion within corona is permissible.

Rakov(2003) in his lightning return stroke engineering models had examined the behaviour of the azimuthal induced magnetic field born out of CG lightning. It was noted by him that the magnetic field might be experienced even several kilometres away from the point of CG lightning. Interestingly the induced azimuthal magnetic field at closure distances (< 50km) exhibit typical hump behaviour just short of 50 µs after originating due the dominant magnetostatic (or induction) component of the total magnetic field. Fig. 1 by Lin et al.(1979) shows the hump in magnetic field.

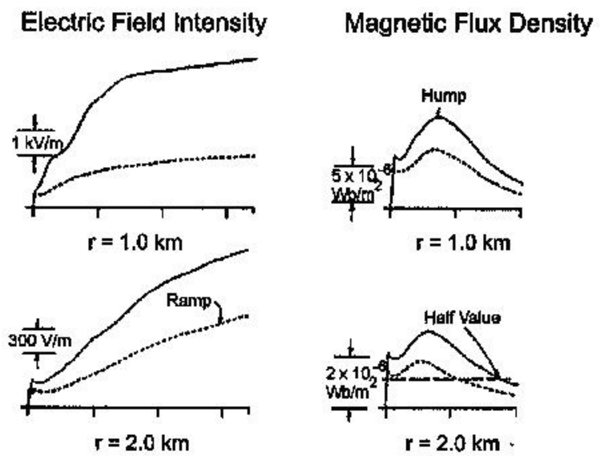
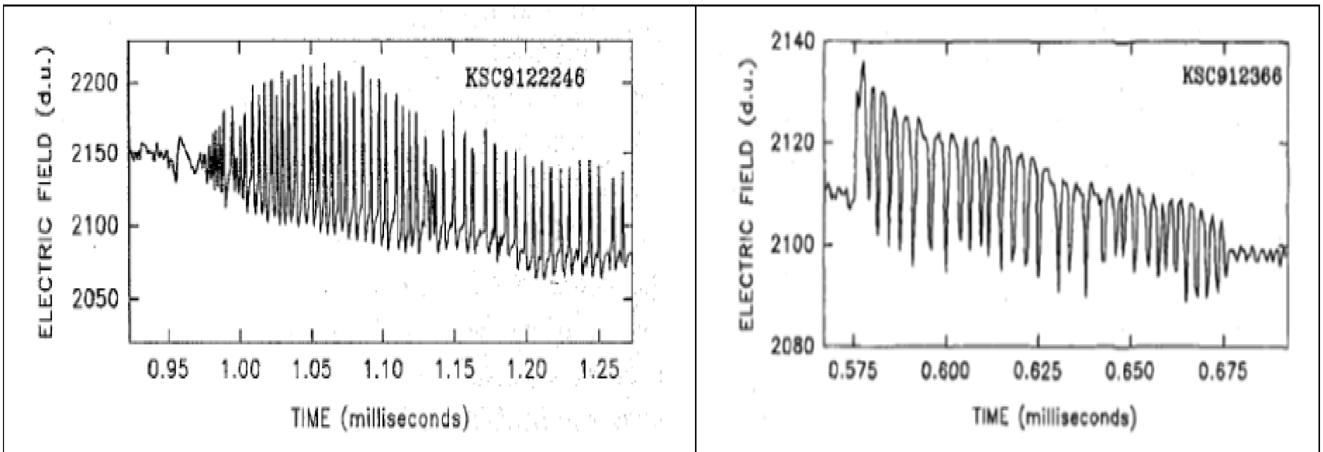


Figure 1: Typical model of electric field intensity (left column) and azimuthal magnetic density (right column) after CG lightning; waveforms for the first(solid line) and subsequent (dashed line) return strokes at a distance of 1 and 2 km{ adapted from Lin et al.(1979)}. 10^{-6} W_b/m^2 = µT.

Without further analysis on this aspect (which is beyond the objective of present study) it is adopted that typically due to CG lightning (as by Lin et al.1979) azimuthal magnetic field at a kilometre away experiences strengthening of magnetic flux density to almost double the initial value within 50 µs and it decays thereafter. Also since as per the Ampere's law the magnetic field (B) = µJA/(2πr){B=magnetic field, µ=magnetic permeability (≈constant), J=electric current density(hence forth used as 'current' in this paper), A= area of current channel and r = distance from the CG channel} is inversely proportional to the distance from the point of CG lightning. It may, therefore, be presumed that under similar conditions magnetic flux density at 50m would be initially 100 µT which will strengthen to 200µT within 50µs before decay. Also for any fixed point the magnetic field (B) is directly proportional to the current (J). Since typical strength of current in negative CG lightning is an order weaker than that in positive



(a) Microsecond-scale electric pulse in CG-lightning. Time scale is 50μs per division.

(b) Microsecond-scale electric pulse in CG-lightning. Time scale is 25μs per division.

Figure 2: Showing fluctuation of electric field on microscale time axis in CG and IC lightning. d.u. denotes digitizer units. Adopted from Rakov et al.(1996).

hence the typical strength of magnetic field in negative CG lightning at any fixed point may also be assumed an order weaker than that during positive CG lightning.

3. Induced Magnetic Field Fluctuation in association with in-situ Geomagnetic Field

Muller-Hillbrand (1962) and Krider et al. (1975) had observed that at shorter distances (<50 km) the waves generated by the 'lightning-bursts' in the electric field induces corresponding spontaneous variability in associated magnetic field during IC and CG lightning flashes though they may not be identical. At distances beyond about 50 km, most lightning electric and magnetic fields are essentially pure radiation fields and hence are identical. In case of CG lightning they noted that magnetic field pulse amplitudes were an order smaller than that by the return strokes. Transport of charges during lightning as 'bursting of pulses form' can be attributed to corresponding induced magnetic field fluctuation. The induced magnetic field lies over the in-situ geomagnetic field. The average typical duration of each burst is of 100-400 µs duration and average interpulse interval is 6.1-7.3 µs. At 50 km the magnetic field was 5 nWb/m^2 (Krider et al.1975). Rakov et al.(1996) had observed that pulse burst tend to occur towards the end of intracloud discharges, where K-changes occur and wave shape of the pulses are similar to that produced by stepped leader process. Microsecond-scale pulse burst in CG lightning on expanded time scale (50µs per division) are shown in Figure 2 (a) and that in case of IC lightning on the expanded time scale (25µs per division) are shown in Figure 2 (b).

Each stroke of current induces flash of magnetic field as per the Faraday's law (Uman 2011). Unfortunately literature is totally devoid of any high frequency magnetogram data during tornadogenesis. Rossow (1970) couldn't capture any ultrahigh magnetic fluctuations with his normal coarse measurements of magnetic field data. Zrnić (1976) had collected normal magnetograms data of two geophysical observatories at distances less than 12 km from tornado touchdowns but 1 sec interval data was not able to resolve high frequency fluctuations of magnetic field in microseconds. Rakov and Uman(1990) have plotted the histogram of 516 interstroke intervals in 132 flashes over florida and New Maxico for negative CG lightning. The time interval between successive return strokes in a flash is usually several tens of milliseconds to many hundreds of milliseconds if long continuous current is involved. Modal values fall between 48 to 64 milliseconds implying that the angular frequency (ω) is mostly observed from 98 to 130 s^{-1} ; though it could range from 12 to 1570 s^{-1}**.** Upward connecting leader (UCL) can be 10-70 m in general and could be up to km in high altitude structure like tower or skyscraper called upward positive leader (UPL). Nag et al. (2009b) had observed that during the initial breakdown process also called preliminary breakdown pulses (PB pulses) 26% of the pulses in the 12 cloud discharges and 22% of the pulses in the 12 cloud-to-ground discharges had total durations less than 1 μs. Hence initially the angular frequency (ω) of corresponding magnetic field fluctuation values could be as much high as ≈ $O(10^7)$. Though average ω could be ≈ 10^3 to 10^5 (Raysaha et al., 2011). Intracloud, positive and negative CG-lightning all exhibit microsecond scale waveforms.

The continuing current which is typically lasting for tens to hundreds of milliseconds, is usually defined as the relatively low-level current of typically tens to hundreds of amperes which immediately follows a return stroke, in the same channel to ground. It is the quasistationary arc between the cloud charge source and ground along the path created by the preceding leader–return-stroke sequence or sequences. Relatively shorter perturbations in the continuing current that typically last for a few milliseconds or less, are called M-components. The contuing-current in the positive lightning is relatively more steady than the negative lightning and it has higher period's wave forms i.e. of the order of milliseconds or hundreds of microseconds. It may be caused by the M-component mode of the charge transfer. After the first return stroke subsequent strokes occur after the cessation of current flow to ground. For negative CG lightning return stroke current typically rise to peak value of 10 to 15 kA in less than a microsecond and decays to half –peak value in a few tens of microseconds. Therefore, broadly speaking, at any time during the stroke, over the background of continuing current or over its rudimentary the microscale and millimiter scale oscillations are superimposed during CG-lightning – may it be dart-lightning or multiple stroke lightning. Also as per Ampere's law the magnetic field (B) = $\mu JA/(2\pi r)$ hence corresponding fluctuations in induced magnetic field are expected. The in-situ geomagnetic field which could be assumed to be nearly-parallel to earth surface and in straight line(preferentially valid over latitudes away from the magnetic poles approximately between ≈ 60^0N and 60^0S) further complicates the behaviour. Earth's magnetic field at its surface ranges from 25 to 65 μT. Undoubtedly therefore, there could be countless variability in oscillatory fluctuations in resultant magnetic field. For theoretical study we take a simple periodic function to glean seminal hint that helps surmise qualitative effects in all other possible complex forms in nature; as it would drastically vary quantitatively on case to case basis of lightning strokes. The study becomes more significant within corona-envelop regime within skin-depth where magneto-hydrodynamical examination of atmospheric flow analyses are permissible.

With the backdrop of foregoing discussion if we assume that during any instance the lightning current J is summation of its oscillatory-current [$(J_0$Cos (ωt)] and rudimentary continuing-current(J_B) where $J_0 > J_B$ then correspondingly we can also assume horizontal magnetic field (B) as $B = B_0\text{Cos}(\omega t) + B_B$**.** It is to be noted that the expression is resultant of induced concentric circular magnetic field in perpendicular plane around the line of flow of electric charges. The horizontal component of the fluctuating concentric circles of magnetic field would create corresponding disturbances over the in situ bed of horizontal geomagnetic field(G_m) in parallel straight lines by IC and CG lightning. Hence resultant fluctuations in the geomagnetic field may be formulated as $B = B_0\text{Cos}(\omega t) + B_{Bm}$ (where $B_{Bm} = B_B + G_m$; a vectorial addition). For example the overlay of the geomagnetic and induced magnetic field by negative CG-lightning is exhibited in Fig. 3. The interference of the two magnetic fields, therefore, is expected to create microsecond-scale and submicrosecond-scale fluctuations in geomagnetic field.

The formulation finds its direct application while incorporating Lorentz force in the momentum equation within corona envelope during the solution of magnetohydrodynamic equations. Note that along the line AA' the directions of the two fields are diagonally opposite and over AC they are aligned together. Hence optimum fluctuation is expected at along AA' and minimum along AC. Nevertheless over entire region wherever the two magnetic fields interfere the in-situ geomagnetic field will experience fluctuations at corresponding levels within cloud and below the cloud, due to IC and CG lightning.

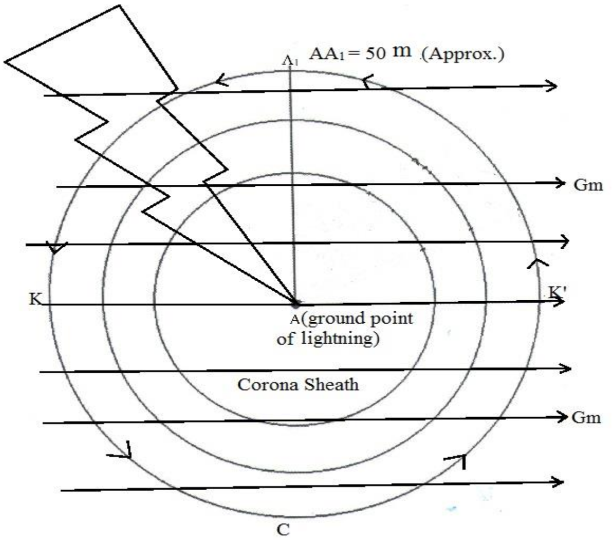


Figure 3: Assuming Negative CG lightning landing at point A then current J is vertically out of plane of paper. Induced circular magnetic field will be anticlockwise. It will be laid over the in-situ horizontal geomagnetic field(G_m) in straight lines. The equation $B = B_0Cos(\omega t) + B_{Bm}$ (where $B_{Bm} = B_B + G_m$; a vectorial addition) would be primarily valid over the line AA' and also to some extent over AC.

4. Conclusions

(i) Lightning current J can be considered as summation of its oscillatory-current [($J_0Cos(\omega t)$)] and rudimentary continuing-current(J_B) similarly one can also express horizontal component of induced magnetic field (B) as $B = B_0Cos(\omega t) + B_B$ **.** Angular frequency (ω) of corresponding magnetic field fluctuation values could be as high as ≈ O(10^7). Though average ω could be ≈ 10^3 to 10^5. As the in-situ geomagnetic field(G_m) interferes with the lightning's magnetic field hence it also spontaneously experiences fluctuations of the same order. It can be formulated by the equation $B = B_0Cos(\omega t) + B_{Bm}$ (where $B_{Bm} = B_B + G_m$; a vectorial addition).

(ii) There will be a diameter-axis for the azimuthal induced magnetic field concentric circles, interfering with the in-situ geomagnetic field, where optimum fluctuations (maximum in one half and minimum in other half) will be experienced.

(iii) Wherever the two magnetic fields interfere within the entire circular region the in-situ geomagnetic field will experience submicrosecond and microsecond fluctuations at corresponding levels within cloud as well as below the cloud till ground level, due to IC and CG lightnings.

(iv) Within the corona envelope the Lorentz force will act as one of the body forces hence magnetohydrodynamical equation has to be solved for atmospheric flow analysis.

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